

Escape over a fluctuating barrier with additive and multiplicative noise

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The mean first passage time (MFPT) over the fluctuating potential barrier is investigated in the presence of additive and multiplicative noises. It is shown that the MFPT over the fluctuating potential barrier displays a resonant activation (RA). The effect of the additive and multiplicative noises and the correlation between them on the RA is that the additive and multiplicative noises can weaken the RA; but the correlation between them can enhance it. The susceptibility of the RA to the multiplicative noise is far larger than that to the additive one. In addition, we find that the transition rate (i.e., the inverse of the MFPT) over the fluctuating potential barrier can be suppressed by the positive correlation and show a minimum as the function of the noises' strengths. [S1063-651X(99)04912-0]

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I. INTRODUCTION

Recently the conventional problems of the escape over the fluctuating potential barrier have attracted a great deal of attention [1–13]. It was shown that the mean first passage time (MFPT) of a particle driven by additive noises over a fluctuating potential barrier exhibits a minimum as a function of the flipping rate of the fluctuating potential barrier [1–12] (or the transition rate of the dichotomous noise [13]). This phenomenon is called “resonant activation,” and was identified by Doering and Gadoua [1] and further studied by a number of other authors [2–13].

Earlier studies of activation of MFPT over fluctuating potentials were restricted to limiting cases, i.e., slow [14] or fast [14,15] barrier fluctuations, or small fluctuations [16]. Owing to using approximate treatments in Refs. [14–16], the resonant activation cannot be observed. Recently in Refs. [1–13], the authors reported results concerning the escape time (i.e., MFPT) over a fluctuating potential in the absence of approximate treatments as in Refs. [14–16]. They revealed the resonant activation (RA) of MFPT over the fluctuating potential barrier.

However, all of the above work for the RA of the MFPT over a fluctuating potential barrier has concentrated on the case where the fluctuating potential barrier is driven by additive noise. One unavoidably wants to ask the question, i.e., if the fluctuating potential barrier is driven by additive and multiplicative noises simultaneously, how is the situation? In addition, in recent years it has been discovered that in systems driven by both additive and multiplicative noises, the two noises can be correlated [17–19], and the correlation is able to change the steady properties of the systems greatly [20–26]. Nevertheless, how the correlation between additive and multiplicative noises alters the activation process is still an interesting and unexposed problem. In this paper we will study the escape time (i.e., MFPT) over the fluctuating potential barrier in the presence of additive and multiplicative noises (between which there is correlation). Here it must be stressed that the multiplicative noise in this paper is different

from the one in Refs. [11,12], etc., the multiplicative noise which appeared in Refs. [11,12], etc., is used to cause the nonfluctuating potential barrier to fluctuate, while the multiplicative noise in this paper is not used to do so.

II. MODEL AND ITS MASTER EQUATION

We consider a model whose Langevin equation is (in dimensionless form)

$$\dot{x} = -\frac{\partial}{\partial x} U(x,t) - \xi(t) \frac{\partial}{\partial x} U(x,t) + \eta(t), \quad (1)$$

where $\xi(t)$ (the multiplicative noise) and $\eta(t)$ (the additive noise) represent the Gaussian white noises. In general, we express the influence of the internal fluctuation on the system as additive noise and the effect of the external environmental fluctuation on the system as multiplicative noise [25–29]. Here we assume that the external environmental fluctuation can influence the internal fluctuation. Because of the influence of the external environmental fluctuation on the internal fluctuation, the additive and multiplicative noises are not independent (there is correlation between them). The statistical properties of $\xi(t)$ and $\eta(t)$ are $\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0$, $\langle \xi(t) \xi(t') \rangle = 2D_2 \delta(t-t')$, $\langle \eta(t) \eta(t') \rangle = 2D_1 \delta(t-t')$, and $\langle \xi(t) \eta(t') \rangle = 2\lambda \sqrt{D_1 D_2} \delta(t-t')$ ($-1 \leq \lambda \leq 1$). $U(x,t)$ is a fluctuating potential barrier which satisfies

$$U(x,t) = U(x) + u(x,t), \quad (2)$$

here the potential at any x fluctuates symmetrically around $U(x)$. $u(x,t)$ can take the values $+\Delta u(x)$ and $-\Delta u(x)$. In Fig. 1 we plot $U(x,t)$ in the case of dimensionless form. We see that the force $F = -\partial U(x,t)/\partial x$ now fluctuates between $F_1^+ = -E_1/\alpha$ ($E_1 = E + \Delta E$) and $F_1^- = -E_2/\alpha$ ($E_2 = E - \Delta E$) on the interval $(0, \alpha)$, and between $F_2^+ = E_1/(1-\alpha)$ and $F_2^- = E_2/(1-\alpha)$ on the interval $(\alpha, 1)$. For convenience,

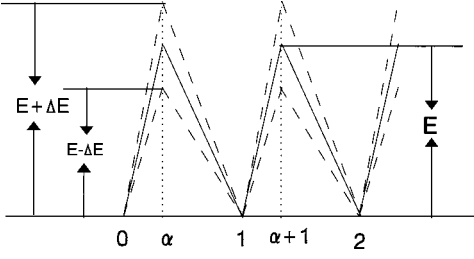


FIG. 1. The fluctuating potential barrier $U(x,t)$ (dashed), which has two configurations, i.e., $U(x) + \Delta u(x)$ (+ configuration) and $U(x) - \Delta u(x)$ (- configuration). The flipping rate of the fluctuating potential barrier between the two configurations is γ . The solid line corresponds to $U(x)$.

below we take $\alpha = 1/2$. The flipping rate of the fluctuating potential barrier is γ . Note that we have $\Delta u(0) = \Delta u(1) = 0$ and $U(x,t) = U(x+1,t)$.

Going from the Langevin equation (1) to the master equations [23,25,30,31] for the probability density distribution we find

$$\frac{\partial}{\partial t} \begin{pmatrix} P_i^+(x,t) \\ P_i^-(x,t) \end{pmatrix} = \begin{pmatrix} G_i^+ & \gamma \\ \gamma & G_i^- \end{pmatrix} \begin{pmatrix} P_i^+(x,t) \\ P_i^-(x,t) \end{pmatrix}, \quad (3)$$

where $G_i^+ = -\gamma - F_i^+ \partial_x + [D_1(1-\lambda^2) + D_2(F_i^+ + \lambda\sqrt{D_1/D_2})^2] \partial_x^2$ and $G_i^- = -\gamma - F_i^- \partial_x + [D_1(1-\lambda^2) + D_2(F_i^- + \lambda\sqrt{D_1/D_2})^2] \partial_x^2$. $i=1$ represents the system on the interval $(0,1/2)$ and $i=2$ represents the system on $(1/2,1)$. The quantities $P^+(x,t)$ and $P^-(x,t)$ are the probabilities at any time t to find the barrier at the + or - configuration, respectively, and the particle at position x . We start with the particle at the bottom ($x=0$). So the initial condition is $\sum_{i=1}^2 P_i(x,0) = \delta(x)$. The boundary conditions for the reflecting ($x=0$) and absorbing ($x=1/2$) boundary, respectively, are $\partial_x P_i(x,t)|_{x=0} = 0$ and $P_i(x,t)|_{x=1/2} = 0$.

III. MEAN FIRST PASSAGE TIME

The equations of MFPT for Eqs. (3) are (see the Appendix)

$$\begin{aligned} & \{-\gamma - 2E_1 \partial_x + [D_1(1-\lambda^2) \\ & + D_2(-2E_1 + \lambda\sqrt{D_1/D_2})^2] \partial_x^2\} T_1 + \gamma T_2 + 1 = 0, \\ & \{-\gamma - 2E_2 \partial_x + [D_1(1-\lambda^2) \\ & + D_2(-2E_2 + \lambda\sqrt{D_1/D_2})^2] \partial_x^2\} T_2 + \gamma T_1 + 1 = 0. \end{aligned} \quad (4)$$

Here the reflecting boundary condition is $\partial_x T_i(0) = 0$, and the absorbing boundary condition is $T_i(1/2) = 0$ ($i=1,2$). The MFPT for a particle over the fluctuating barrier that starts at the bottom ($x=0$) is $T = \sum_{i=1}^2 T_i(0)$. Taking $\partial_x T_i = s_i$ ($i=1,2$), from Eq. (4) we can obtain

$$\partial_x \begin{pmatrix} T_1 \\ s_1 \\ T_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{\gamma}{A_1} & \frac{2E_1}{A_1} & -\frac{\gamma}{A_1} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\gamma}{A_2} & 0 & \frac{\gamma}{A_2} & \frac{2E_2}{A_2} \end{pmatrix} \begin{pmatrix} T_1 \\ s_1 \\ T_2 \\ s_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1/A_1 \\ 0 \\ -1/A_2 \end{pmatrix}, \quad (5)$$

where $A_1 = D_1(1-\lambda^2) + D_2(-2E_1 + \lambda\sqrt{D_1/D_2})^2$, and $A_2 = D_1(1-\lambda^2) + D_2(-2E_2 + \lambda\sqrt{D_1/D_2})^2$.

A. Case of $E_1 + E_2 = 0$

In general, one cannot obtain the exact expression of the MFPT. However, in the case where the midpoint of the barrier fluctuates between $\pm E$ (that is $E_1 + E_2 = 0$, and $E_1 = -E_2 = E$) it is simple enough to summarize analytically. Now the MFPT for a particle over a fluctuating potential barrier is, explicitly,

$$\begin{aligned} T = & c_1 \left(2 + \frac{2E}{\gamma} r_1 - \frac{A_1}{\gamma} r_1^2 \right) + c_2 \left(2 + \frac{2E}{\gamma} r_2 - \frac{A_1}{\gamma} r_2^2 \right) + 2c_4 \\ & + \frac{2E}{\gamma} c_3 + \frac{2A_1}{4E^2 + \gamma(A_1 + A_2)} - \frac{1}{\gamma}, \end{aligned} \quad (6)$$

where $r_{1,2} = [2EA_2 - 2EA_1 \pm \sqrt{(EA_2 + EA_1)^2 + \gamma A_1 A_2 (A_1 + A_2)}] / (A_1 A_2)$, $c_1 = (k_2 k_3' - k_3 k_2') / (k_1 k_2' - k_2 k_1')$, $c_2 = (-k_1 c_1 - k_3) / k_2$, $c_3 = -c_1 r_1 - c_2 r_2$, $c_4 = -c_1 \exp(r_1) - c_2 \exp(r_2) - c_3 - \gamma[-4E^2 - \gamma(A_1 + A_2)]$, with $k_1 = (2Er_1 / \gamma - A_1 r_1^2 / \gamma) r_1$, $k_2 = (2Er_2 \gamma - A_1 r_2^2 / \gamma) r_2$, $k_3 = 4E / [-4E^2 - \gamma(A_1 + A_2)]$, $k_1' = -2Er_1 / \gamma + (2Er_1 / \gamma - A_1 r_1^2 / \gamma) \exp(r_1)$, $k_2' = -2Er_2 / \gamma + (2Er_2 / \gamma - A_1 r_2^2 / \gamma) \exp(r_2)$, and $k_3' = 4E / [-4E^2 - \gamma(A_1 + A_2)] - 2A_1 / [-4E^2 - \gamma(A_1 + A_2)] - 1/\gamma$.

B. Case of $E_1 + E_2 \neq 0$

When $E_1 + E_2 \neq 0$ we cannot analytically get the exact expression of the MFPT. Below we give the derivation of the expression of the MFPT for numerical simulation. By numerical simulation and analysis we can find that when $E_1 + E_2 \neq 0$ the matrix of the homogeneous part about T_i and s_i ($i=1,2$) in Eq. (5) has three nonzero real independent eigenvalues and a zero eigenvalue. The general solution of Eq. (5) ($E_1 + E_2 \neq 0$) are $s_i = \sum_{j=1}^3 A_j^{(i)} \exp(\lambda_j x) + A_4^{(i)} + A_5^{(i)} x$ and $T_i = \sum_{j=1}^3 B_j^{(i)} \exp(\lambda_j x) + B_4^{(i)} + B_5^{(i)} x$, where $i=1,2$, λ_j ($j=1,2,3$) are the above-mentioned nonzero eigenvalues. Substituting s_i and T_i into $\partial_x T_i = s_i$ we can obtain $B_j^{(i)} = A_j^{(i)} / \lambda_j$, $A_5^{(i)} = 0$, and $B_5^{(i)} = A_4^{(i)}$. So we have

$$s_i = \sum_{j=1}^3 A_j^{(i)} \exp(\lambda_j x) + A_4^{(i)}, \quad (7)$$

$$T_i = \sum_{j=1}^3 \frac{A_j^{(i)}}{\lambda_j} \exp(\lambda_j x) + B_4^{(i)} + A_4^{(i)} x.$$

Substituting Eq. (7) into Eq. (5) and using the comparing-coefficient method, we obtain $A_4^{(i)} = 2/(E_1 + E_2)$, $B_4^{(i)} = B_4^{(1)} + F_i$, and $A_j^{(i)} = k_j^{(i)} A_j^{(1)}$ ($i = 1, 2$, and $j = 1, 2, 3$) with $F_1 = 0$, $F_2 = (E_1 + 3E_2)/\gamma(E_1 + E_2)$, $k_j^{(1)} = 1$, $k_j^{(2)} = 1 + 2E_1 \lambda_j / \gamma - A_1 \lambda_j^2 / \gamma$. Substituting Eq. (7) into the boundary conditions $T_i(1/2) = 0$ and $s_i(0) = 0$ [note $A_4^{(i)} = 2/(E_1 + E_2)$, $B_4^{(i)} = B_4^{(1)} + F_i$, and $A_j^{(i)} = k_j^{(i)} A_j^{(1)}$], we can obtain a linear algebraic system for $A_j^{(1)}$ ($j = 1, 2, 3$) and $B_4^{(1)}$. From the linear algebraic equations of this algebraic system we can derive $A_j^{(1)}$ and $B_4^{(1)}$. The MFPT for a particle over the fluctuating barrier is

$$T = \sum_{i=1}^2 T_i(0) = \sum_{i=1}^2 \sum_{j=1}^3 \frac{k_j^{(i)}}{\lambda_j} A_j^{(1)} + 2B_4^{(1)} + \sum_{i=1}^2 F_i. \quad (8)$$

IV. CONCLUSION AND DISCUSSION

If we do not consider the multiplicative noise, Eq. (1) becomes the model studied by Doering and Gadoua [1], and by Bier and Astumian [2]. Doering, Gadoua, Astumian, and Bier have identified the RA of the MFPT for a particle over a fluctuating potential barrier for Eq. (1) in the absence of the multiplicative noise. For the stochastic model (1), we will ask the following questions. Is there the RA of the MFPT for a particle over the fluctuating potential barrier? If there is the RA, how do the multiplicative noise and the correlation between the additive and multiplicative noises affect it? In order to settle the two questions, we plot Figs. 2 and 3 according to Eqs. (6) and (8). In Figs. 2 and 3, the \ln of the MFPT versus the \ln of the flipping rate γ of the fluctuating potential barrier is plotted when $E_1 + E_2 = 0$ and $E_1 + E_2 \neq 0$, respectively. From these figures, one can find that there is a RA for the MFPT over the fluctuating potential barrier. A reason for this RA happening here is given below. The resonance in Figs. 2 and 3 occurs when the crossing takes place with the fluctuation potential barrier most likely in $E = \min(E_1, E_2)$ configuration (i.e., the ‘‘down’’ configuration). Now the MFPT has a local minimum for the fluctuation potential barrier transition rate on the order of the inverse of the time required to cross the fluctuation barrier with the fluctuation potential barrier in $E = \min(E_1, E_2)$ configuration. In Figs. 2 and 3 we plot the corresponding points where the transition time equals the MFPT over the fluctuating barrier with the potential barrier in $E = \min(E_1, E_2)$ configuration. It is clear that this accords with the above reason for the RA happening in Figs. 2 and 3. Moreover, from these figures we can find that (1) with increasing the noises’ (additive noise and multiplicative noise) strength, the RA becomes more and more indistinct, i.e., the noises can weaken the RA; (2) with increase of the correlation between the additive and multiplicative noises, the RA becomes more and more distinct, i.e., the correlation can enhance the RA; (3) the susceptibility of the RA to the multiplicative noise is far larger than that to the additive one.

In addition, we should investigate the activation of the MFPT as the function of the noises’ strength for different

values of the correlation between the additive and multiplicative noises. We find that when $E_1 + E_2 \neq 0$, the escape rate (i.e., the reciprocal value of the MFPT) over the fluctuating potential barrier can be suppressed by the positive correlation and show a minimum as the function of the two noises’ strengths. This phenomenon has been reported by Madureira, Hånggi, and Wio [32], and Hai-xiang Fu, Li Cao, and Da-jin Wu [33]. They named this phenomenon ‘‘giant suppression of the activation rate’’ (GS). In Figs. 4(a) and 4(b) we plot the \ln of the MFPT versus the additive noise strength in the case of $E_1 + E_2 = 0$ and $E_1 + E_2 \neq 0$, respectively. Figure 4(a) shows that when $E_1 + E_2 = 0$ the MFPT versus the additive noise strength is monotonous (no GS exists); and the MFPT becomes larger and larger with increasing the value of the correlation. Figure 4(b) shows that the MFPT curve for positive correlation exhibits a peak value (i.e., GS exists), while curves for noncorrelation and negative correlation do not, and the larger the correlation strength is, the higher the peak becomes. Now the positive correlation becomes more suppressive on the activation as the correlation grows, which is exactly the main conclusion of Refs. [32,33]. This is because when the correlation is positive the instantaneous barrier can be lifted up (the negative correlation case is just the contrary).

Below we consider the case when $-\xi(t)\partial_x U(x, t)$ in Eq. (1) is $-\xi(t)\partial_x \tilde{U}(x, t)$, in which $\tilde{U}(x, t)$ is a fluctuating barrier with the flipping rate $\tilde{\gamma}$. $-\partial_x \tilde{U}(x, t)$ takes the values $-\tilde{E}_1$ and $-\tilde{E}_2$ on the interval $(0, 1/2)$, and \tilde{E}_1 and \tilde{E}_2 on the interval $(1/2, 1)$. Then Eq. (1) becomes

$$\dot{x} = -\partial_x U(x, t) - \xi(t)\partial_x \tilde{U}(x, t) + \eta(t). \quad (9)$$

It is clear that Eq. (1) is a special case for Eq. (9) in the case of $\tilde{\gamma} = \gamma$, $\tilde{E}_1 = E_1$ and $\tilde{E}_2 = E_2$. Further study shows that the stochastic system with stochastic differential equation (9) has the same phenomenon as shown in Figs. 2, 3, and 4 [i.e., (1) there is RA for MFPT; (2) the additive and multiplicative noises can weaken the RA, but the correlation between them can enhance the RA; (3) the susceptibility of the RA to the multiplicative noise is far larger than that to the additive one; (4) there is the GS]. In order to avoid unnecessary repetition we do not present the figures that are basically similar to Figs. 2, 3, and 4. In Ref. [13], we study the escape time over a fluctuating barrier in the presence of a dichotomous noise and a Gaussian white noise. It is shown that the mean first passage time (MFPT) over the fluctuating barrier displays two RA’s. One is the RA of the MFPT as a function of the flipping rate of the fluctuating potential barrier; the other is the RA of the MFPT as a function of the transition rate of the dichotomous noise. As for the model (9), there is only the RA for the MFPT over the fluctuating potential barrier as a function of the flipping rate γ of the fluctuating potential barrier $U(x, t)$, but there is not the RA for the MFPT over the fluctuating potential barrier as a function of the flipping rate $\tilde{\gamma}$ of the fluctuating barrier $\tilde{U}(x, t)$. Here the flipping rate $\tilde{\gamma}$ of the fluctuating barrier $\tilde{U}(x, t)$ has little influence on the MFPT over the fluctuating potential barrier (see Fig. 5). In Fig. 5 we plot the \ln of the MFPT versus the \ln of the flipping rate γ of the fluctuating potential barrier for $E_1 = 5$,

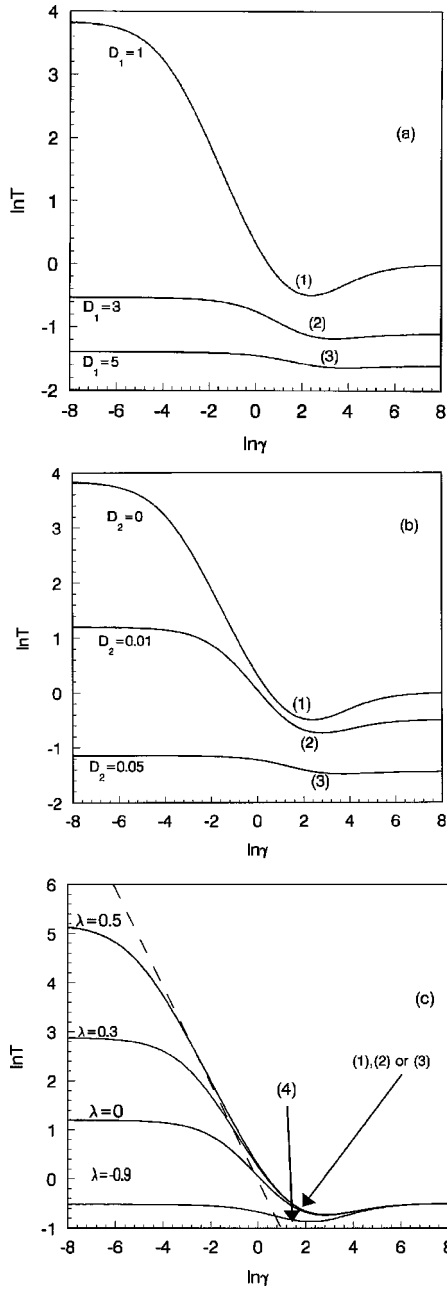


FIG. 2. The \ln of the MFPT versus the \ln of the flipping rate γ of the fluctuating potential barrier when $E_1 + E_2 = 0$. (a) Corresponds to the \ln of the MFPT versus the \ln of γ for different values of the additive noise strength D_1 ($D_1 = 1, 3$ and 5), $E_1 = -E_2 = 4$, $D_2 = 0$, and $\lambda = 0$; (b) corresponds to that for different values of the multiplicative noise strength D_2 ($D_2 = 0, 0.01$, and 0.05), $E_1 = -E_2 = 4$, $D_1 = 1$, and $\lambda = 0$; (c) corresponds to that for different values λ of the correlation between the additive and multiplicative noises ($\lambda = -0.9, 0, 0.1$, and 0.5), $E_1 = -E_2 = 4$, $D_1 = 1$, and $D_2 = 0.01$. The marked points (1), (2), (3), and (4) are the corresponding points where the transition time equals the MFPT over the fluctuating barrier with the fluctuating potential barrier in the “down” configuration. In (c), a dashed line is plotted for the case when γ' (i.e., $1/T$) equals γ .

$E_2 = 7$, $\tilde{E}_1 = -\tilde{E}_2 = 1$, $D_1 = 0$, and $D_2 = 1$. The solid line corresponds to $\tilde{\gamma} = 1000$, and the dashed line to $\tilde{\gamma} = 10$.

The relation of our work to the phenomenon of stochastic resonance should be considered. For the phenomenon of sto-

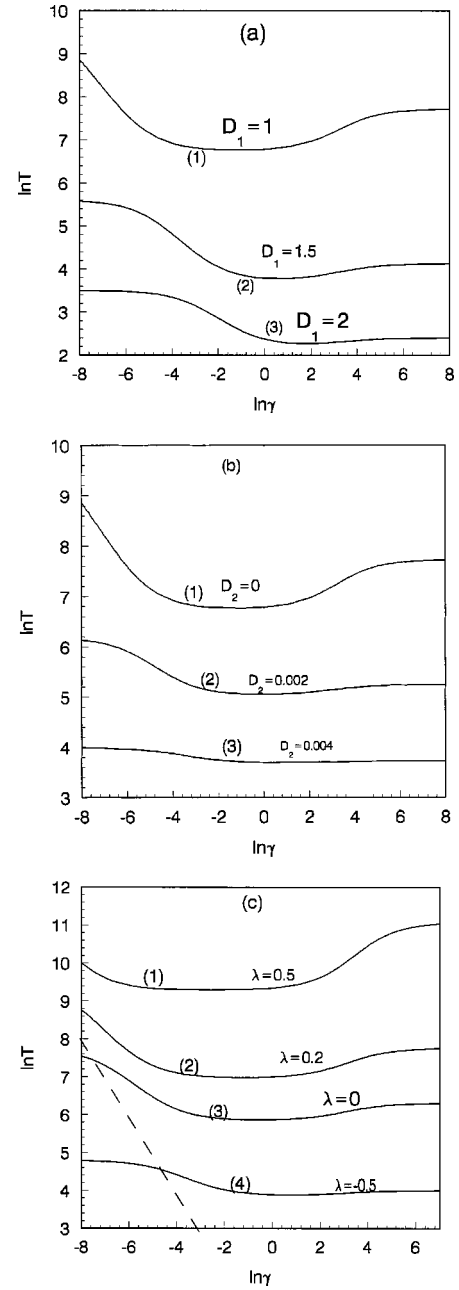


FIG. 3. The \ln of the MFPT versus the \ln of the flipping rate γ of the fluctuating potential barrier when $E_1 + E_2 \neq 0$. (a) Corresponds to the \ln of the MFPT versus the \ln of γ for different values of the additive noise strength D_1 ($D_1 = 1, 1.5$, and 2), $E_1 = 5$, $E_2 = 7$, $D_2 = 0$, and $\lambda = 0$; (b) corresponds to that for different values of the multiplicative noise strength D_2 ($D_2 = 0, 0.002$, and 0.004 , respectively), $E_1 = 5$, $E_2 = 7$, $D_1 = 1$, and $\lambda = 0$; (c) corresponds to that for different values λ of the correlation between the additive and multiplicative noises ($\lambda = -0.5, 0, 0.2$, and 0.5), $E_1 = 5$, $E_2 = 7$, $D_1 = 1$, and $D_2 = 0.001$. The marked points (1), (2), (3), and (4) are the corresponding points where the transition time equals the MFPT over the fluctuating barrier with the fluctuating potential barrier in the “down” configuration. (c) A dashed line is plotted for the case when γ' (i.e., $1/T$) equals γ .

chastic resonance, we know that the response of a nonlinear stochastic system to an inputting signal will be enhanced by the presence of noise and maximized for certain value of the noise's strength. When the frequency of the inputting signal

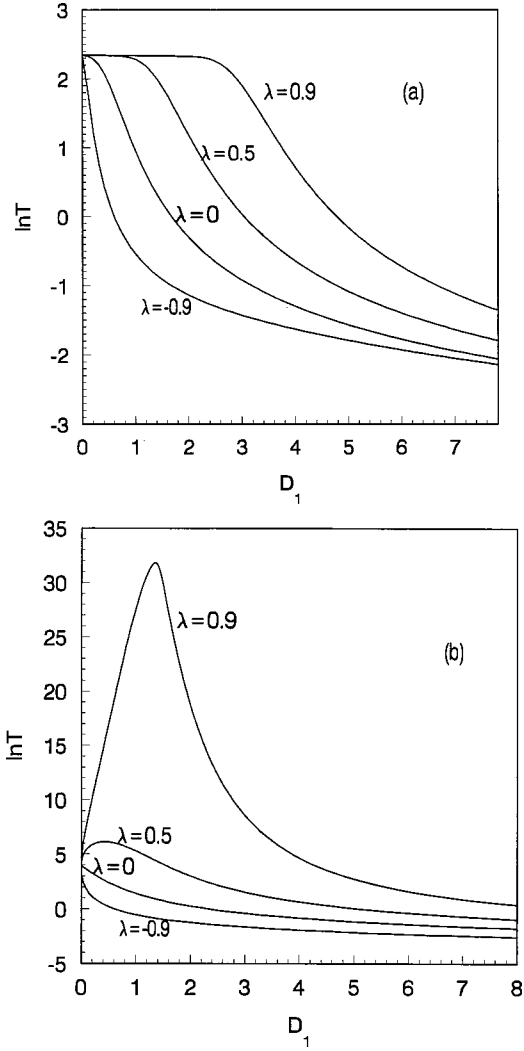


FIG. 4. The \ln of the MFPT versus the additive noise strength for different values of λ ($\lambda = -0.9, 0, 0.5,$ and 0.9) when $E_1 + E_2 = 0$ for $E_1 = -E_2 = 4$, $D_2 = 0.01$, and $\gamma = 0.1$ [(a)], and when $E_1 + E_2 \neq 0$ for $E_1 = 5$, $E_2 = 7$, $D_2 = 0.01$, and $\gamma = 10$ [(b)].

is equal to the intrinsic frequency of the original stochastic system, a phenomenon of resonance will appear. In our paper, the RA and the GS have both the phenomenon of resonance. First, let us analyze the phenomenon of resonance appearing in the RA. For small values γ of the flipping rate of the fluctuating potential barrier, a destructive influence on the asymmetry of the system will be played, so the $\ln(1/T) - \ln\gamma$ response curve will have positive slope. For large γ , a central role will be played in producing coherent motion with increases as γ increases, then the $\ln(1/T) - \ln\gamma$ curve goes down. Thus, finally we can obtain a peaked $\ln(1/T) - \ln\gamma$ curve, at the peak of which a phenomenon of resonance will happen. As for the phenomenon of resonance appearing in the GS, the same analysis can be made as the one in the RA. The intrinsic frequency of the stochastic system studied by us is $\gamma' = 1/T$ in which T is the MFPT over the fluctuating potential barrier. When γ' is equal to the frequency of the fluctuating potential barrier, i.e., the flipping rate γ , no resonance happens. In Figs. 2(c) and 3(c) we plot the lines when γ' is equal to γ (the dashed lines).

Finally, it needs to be explained that, when we calculate

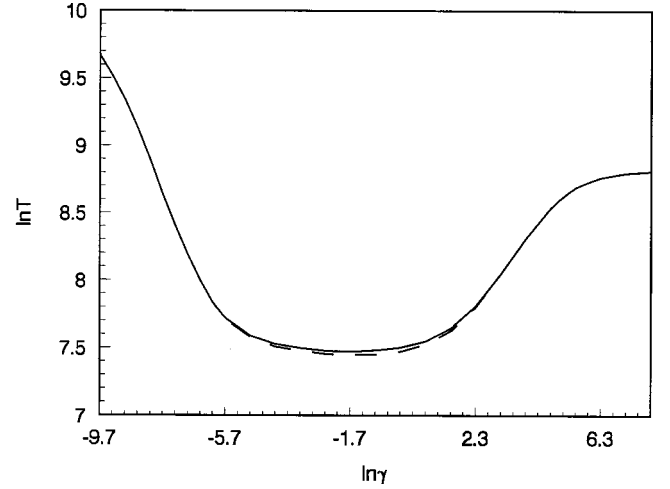


FIG. 5. The \ln of the MFPT versus the \ln of the flipping rate of the fluctuating potential barrier for model (9). $E_1 = 5$, $E_2 = 7$, $\bar{E}_1 = -\bar{E}_2 = 1$, $D_1 = 0$, and $D_2 = 1$. $\tilde{\gamma} = 1000$ (solid) and 10 (dashed), respectively.

the MFPT for a particle over the fluctuating potential barrier [in the dimensionless form, $E \sim 5$ (see the Figs. 1, 2, and 3 in the paper)], we only calculate the MFPT from $x = 0$ to $x = 1/2$, which is because the MFPT from $x = 0$ to $x = 1/2$ is far larger than that from $x = 1/2$ to $x = 1$. In this paper we believe that the particle moves from left to right. When the particle moves from right to left, since we take $\alpha = 1/2$, the MFPT over the fluctuating potential barrier is similar to the one when the particle moves from left to right. (This means that the MFPT over the fluctuating potential barrier is symmetric with response to the direction of barrier crossing.)

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APPENDIX

The backward master equation for master (3) is [30,31]

$$\frac{\partial}{\partial t} \begin{pmatrix} Q_i^+(x,t) \\ Q_i^-(x,t) \end{pmatrix} = \begin{pmatrix} \bar{G}_i^+ & \gamma \\ \gamma & \bar{G}_i^- \end{pmatrix} \begin{pmatrix} P_i^+(x,t) \\ P_i^-(x,t) \end{pmatrix}, \quad (\text{A1})$$

where $\bar{G}_i^+ = -\gamma + F_i^+ \partial_x + [D_1(1-\lambda^2) + D_2(F_i^+ + \lambda\sqrt{D_1/D_2})^2] \partial_x^2$ and $\bar{G}_i^- = -\gamma + F_i^- \partial_x + [D_1(1-\lambda^2) + D_2(F_i^- + \lambda\sqrt{D_1/D_2})^2] \partial_x^2$. $i = 1$ represents the system on the interval $(0, 1/2)$ and $i = 2$ represents the system on $(1/2, 1)$.

The MFPT is defined as [30]

$$T_1(x) = - \int_0^\infty t \partial_t Q_1^+(x,t) dt = \int_0^\infty Q_1^+(x,t) dt,$$

$$T_2(x) = - \int_0^\infty t \partial_t Q_1^-(x, t) dt = \int_0^\infty Q_1^-(x, t) dt, \quad (\text{A2})$$

where we only calculate the MFPT from $x=0$ to $x=1/2$, because the MFPT from $x=0$ to $x=1/2$ is far larger than that from $x=1/2$ to $x=1$.

From Eqs. (A1) and (A2), one obtains the equations of the MFPT:

$$\{-\gamma - E_1 \partial_x + [D_1(1-\lambda^2) + D_2(-E_1 + \lambda \sqrt{D_1/D_2})^2] \partial_x^2\} T_1 + \gamma T_2 + 1 = 0,$$

$$\{-\gamma - E_2 \partial_x + [D_1(1-\lambda^2) + D_2(-E_2 + \lambda \sqrt{D_1/D_2})^2] \partial_x^2\} T_2 + \gamma T_1 + 1 = 0. \quad (\text{A3})$$

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